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WORST CASE GEOMETRIC SCENARIOS FOR GEO-LOCATION DETERMINATION BY A NETWORK OF SATELLITE MOUNTED SENSORS
(An Interim Report)

by

I. Bert Russak

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NAVAL POSTGRADUATE SCHOOL
Monterey, California 93943-5000

RADM Robert C. Chaplin
Superintendent

R. Elster
Provost

This report was prepared for Naval Postgraduate School and funded by Reconnaisance Research, NPS.

This report was prepared by:

I. Bert Russak
Associate Professor
Mathematics Department

Reviewed by:

Michael A. Morgan, Chair
Mathematics Department

Released by:

D. W. Netzer
Associate Provost and
Dean of Research

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I. Bert Russak

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Naval Postgraduate School
Department of Mathematics
1411 Cunningham Road
Monterey, California 93943

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13. ABSTRACT (Maximum 200 words.)

This work deals with worst case geometric scenarios of a system of satellite mounted sensors in determining the location of a signal source on the earth's surface.
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A) OBJECTIVE.

Determination of the location on the earth's surface of an electromagnetic emitter by a single satellite sensor or a network of satellite sensors, is possible if the variables which define the location(s) and orientation(s) of the sensor(s) is known exactly. However if an emitter location is determined according to inaccurate information of sensor(s) location and orientation, then the determination will also be in error.

In this study, we will first handle the case of a single sensor. Networks of two or more sensors will be handled later. The error in the location determination is defined as follows: In figure 1, let S_1 represent the location and orientation of the sensor according to inaccurate information. Also let I_1 represent the emitter location due to that data. Let $S_2, and\ I_2$ be identified as were S_1, I_1 but according to exact data. The error is then the vector difference $I_2 - I_1$. In this study, $I_2 - I_1$ will be approximated by its total differential dI. The formula relating the total differential to the sensor variables will be used to find the absolute maximum(s) of $|dI|$ with respect to changes in the sensor variables.

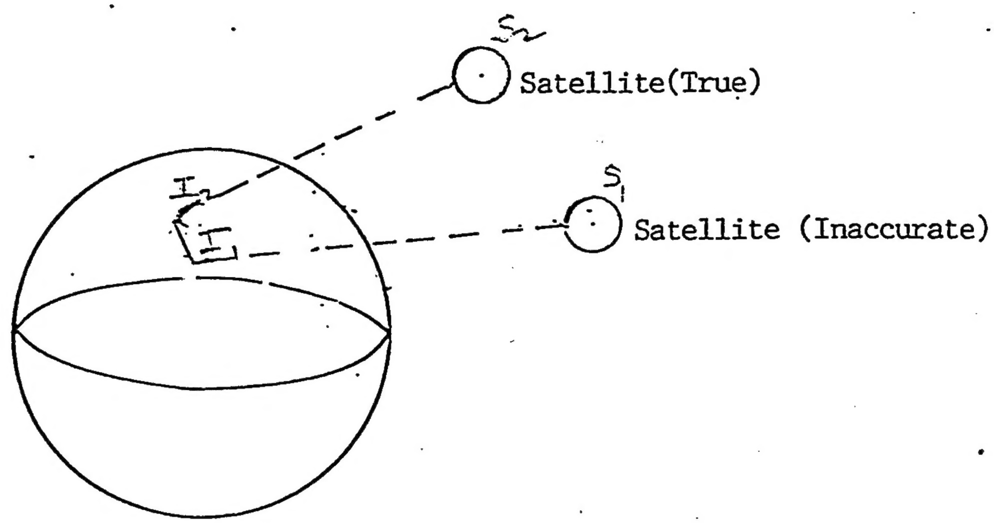

Figure 1. Geometry of the relation between the error $I_2 - I_1$ and the assumed and true satellite positions S_1 *and* S_2 respectively.

Definition of the sensor location and orientation variables

Referring to figure 2, consider the following situation. A satellite sensor system is located at point S at a certain instant of time t_0. The sensor is orientated so that at t_0, it's "sight axis" is along SI which meets the earth's surface at I. At that time the sensor detects a signal from I.

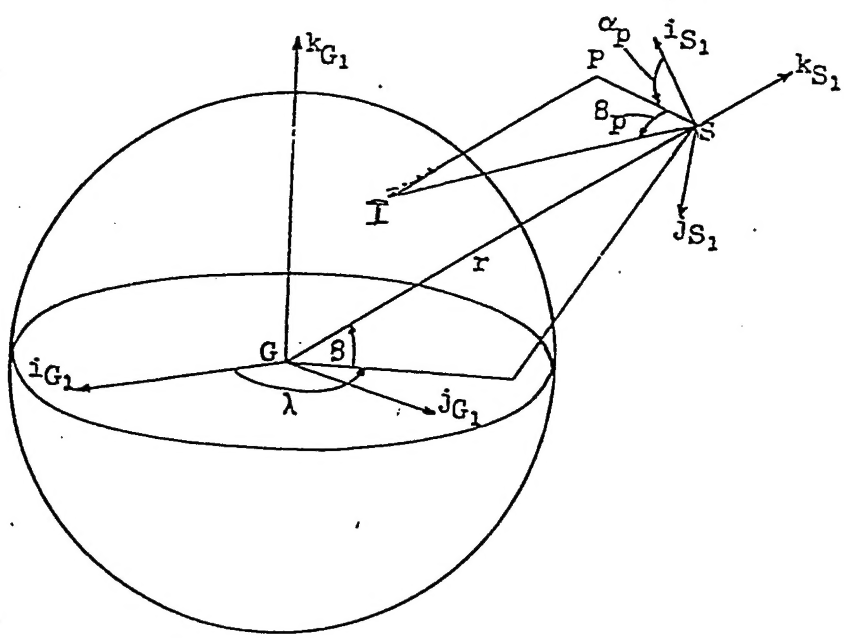

Figure 2. Geometric relationship between the sensor at S and an emitter at I on the earth's surface.

Introduce the inertial geocentric right hand coordinate system $[G_1]$ *with plane* $i_{[G_1]} - j_{[G_1]}$ coincident with the equatorial plane.[a] It will be assumed that system $[G_1]$ is the known basic coordinate system into which all vectors will be resolved in order to compare to other vectors.

The vector GS from geocenter to satellite is described in terms of the three quantities, λ, β *and* $r = r_e + h$ which are measured as indicated in figure 2. The angles α_p *and* β_p then complete the description of the sensor's attitude. These last two angles define the orientation with respect to system $[S_1]$, which is characterized by having origin at S, axis k_{S_1} colinear with GS and axis i_{S_1} pointing North (see figure 2). Thus the five variables $r, \lambda, \beta, \alpha_p,$ *and* β_p are sufficient to describe the location and orientation of the sensor.

[a] System $[G_1]$ has coordinate axes $i_{[G_1]}, j_{[G_1]}, k_{[G_1]}$. This convention is used throughout this paper so that, for example coordinate system $[L_k]$ has coordinate axes $i_{[L_k]}, j_{[L_k]}, k_{[L_k]}$. All coordinate systems will be right hand cartesian. Furthermore the title of any particular coordinate system will indicate its origin of coordinates so that for example, all $[G_j]$ systems will be geocentric, while all $[S_k]$ systems have origin at S. Also, letters in brackets will indicate coordinate systems. Finally, a quantity that has a subscript consisting of a bracketed symbol, will indicate the representation of that quantity in that coordinate system. Thus e.g. $I_{[S_1]}$ *is the representation of* I *in* $[S_1]$ *coordinates.*

ANALYSIS.

The detailed geometric action of the sensor can be thought of as a mapping of a point I on the earth's surface onto an image point on the z-axis of a coordinate system with origin at S as follows: Refer to figure 3 and introduce coordinate system $[S_2]$ which is characterized by having axis $k_{[S_2]}$ opposite to SI and axis $j_{[S_2]}$ parallel to $SP \times SI$ [a].

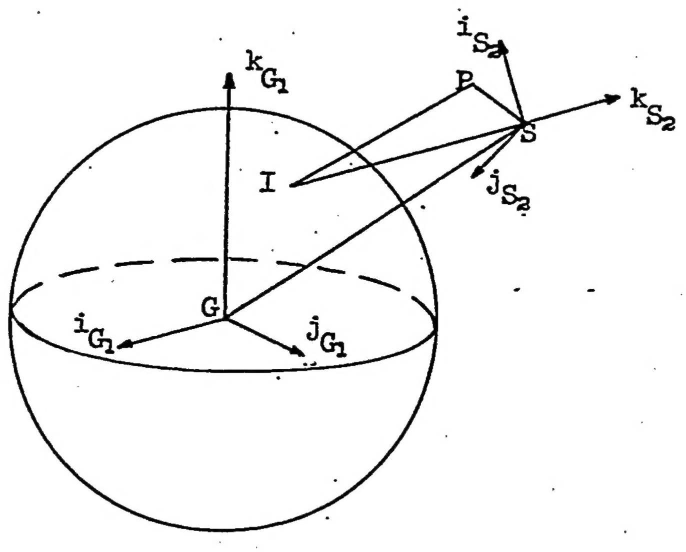

Figure 3. Geometry of Coordinate System $[S_2]$

[a] SP is the projection of SI on the plane $i_{[S_2]}$, $j_{[S_2]}$

Now, knowing the quantities r, λ, β, α_P, β_P, we can determine the location of S and the representation of vectors SI and SP in $[G_1]$ (See figure 2). Then as these last two vectors determine two axes of coordinate system $[S_2]$, it is possible to obtain completely the rotation matrix (M) from $[S_2]$ *to* $[S_G]$[a]. The complete transformation of point "I" is then the result of a translation along IS, a rotation through the matrix $(M)=(m_{ij})$ where $m_{ij}=f_{ij}(\lambda, \beta, \alpha_P, \beta_P)$ and a translation along SG. If now the quantities r, λ, β, α_P, β_P are altered by their respective increments to values r', λ', β', α_P', β_P' (where for example, $r'=r+\Delta r$), S shifts to S' , and the vector SI becomes $S'I'$ (see figure 4). The error $I'-I$ will be approximated by the total differential dI represented in coordinate system $[G_1]$.

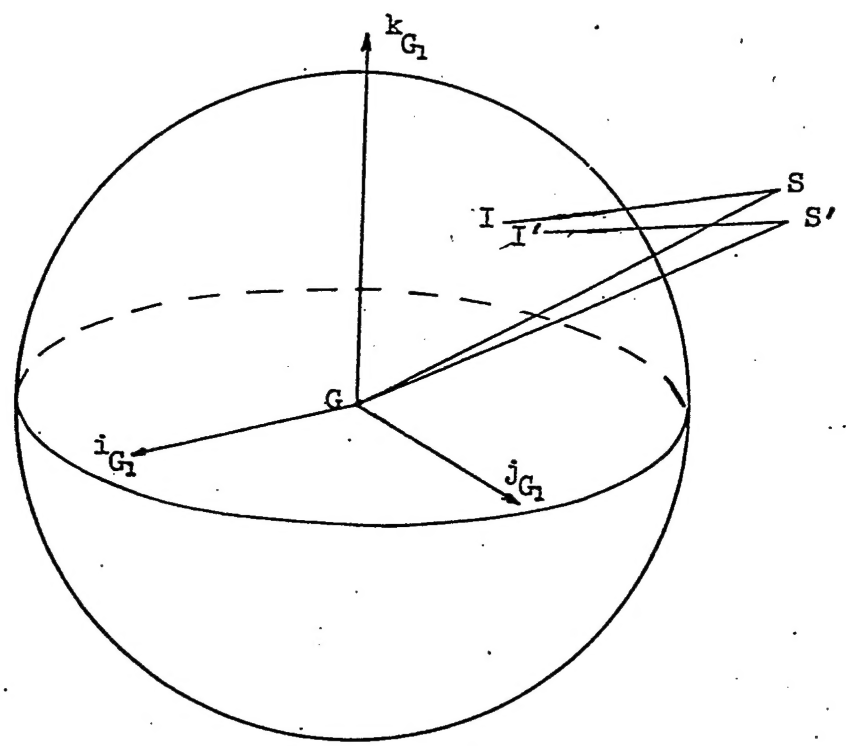

Figure 4. Geometry of the true and inaccurate sensor situations

[a] The system $[S_G]$ is parallel to system $[G_1]$, but with origin at S.

The process of determining the point I on the earth and also its total differential dI will now be described in detail. With S as the origin of the $[S_2]$ coordinates, the $[S_2]$ coordinates of S are of course (0,0,0). We first determine the point I, which is the intersection of SI with the earth's surface. In $[S_2]$ coordinates, the coordinates of I are $(0,0,I_{k_{[S_2]}})$. $I_{k_{[S_2]}}$ is determined in Appendix II. Let (M) be the rotation matrix that maps coordinates in $[S_2]$ to those in $[S_G]$ (introduced previously). (M) is derived in appendix 1. Thus the $[S_G]$ coordinates of I are $I_{[S_G]} = (M)(0,0,I_{k_{[S_2]}})^T$. Finally, a translation by $r_e + h$ along the negative of $k_{[S_1]}$ yields I in $[G_1]$ coordinates. Finally, we compute dI, the total differential of I with respect to the variables that define sensor location and orientation. dI is the representation of the error that we seek.

Thus, given a set of values of

r, λ, β, α_P, β_P, *then the location of the emitter, I is* $I_{[S_2]} = 0i_{[S_2]} + 0j_{[S_2]} - |SI|\,k_{[S_2]}$,

where

$$|SI| = r\sin\beta_p - \sqrt{r_e^{\,2} - (r\cos\beta_p)^2}$$

as shown in Appendix II. Then pre-multiplying $I_{[S_2]}$ by the rotation matrix (M) (derived in Appendix I) yields $I_{[S_G]}$

$$I_{[S_G]} = (M)\; I_{[S_2]}^T$$

as

$$I_{[S_G]} = |SI|\,(m_{13} i_{[S_G]} + m_{23} j_{[S_G]} + m_{33} k_{[S_G]})$$

Now doing a translation along the vector GS (which is along the $k_{[S_1]}$ axis) and using the first equation in Appendix I, we obtain

$$I_{[G_1]} = I_{[S_G]} - |SG|\,(i_{[S_G]}\cos\beta\cos\lambda + j_{[S_G]}\cos\beta\sin\lambda + k_{[S_G]}\sin\beta)$$

which upon substitution from the above can also be written

$$\begin{aligned} I_{[G_1]} \quad = \quad & (m_{13}|SI| - |SG|\cos\beta\cos\lambda)\, i_{[G_1]} + (m_{23}|SI| - |SG|\cos\beta\sin\lambda)\, j_{[G_1]} + \\ & (m_{33}|SI| - |SG|\sin\beta)\, k_{[G_1]} \end{aligned}$$

where in the immediately above relation we have recognized that the coordinates systems $[G_1]$ *and* $[S_G]$ have identical coordinate axes. This constitutes the desired quantity and coordinate system in which to represent it. Next, the total differential dI with respect to the variables $r, \lambda, \beta, \alpha_p, \beta_p$ and the function $|dI|^2$ will be formed The first partial derivatives of $|dI|^2$ will be formed with respect to the variables r, λ, β, α_p, β_p and critical points identified. Domain end-points will also be inspected. From these points, absolute maximum(s) of $|dI|^2$ will be determined.

APPENDIX 1

DERIVATION OF THE (M) MATRIX

In order to derive the (M) matrix it is necessary in what follows to work with coordinate systems having a common origin. Accordingly we have previously introduced the system $[S_G]$ as being parallel to $[G_1]$ but with origin at point S.

By recalling Figure 2 and our definition of system $[S_1]$, it is clear that the representation in coordinate system $[S_G]$ of axis $k_{[S_l]}$ is

$$k_{[S_l]} = i_{[S_G]}\cos\beta\cos\lambda + j_{[S_G]}\cos\beta\sin\lambda + k_{[S_G]}\sin\beta .$$

Furthermore, axis $i_{[s_l]}$ points North from point S and therefore lies in the plane containing axes $k_{[S_l]}$ *and* $k_{[G_l]}$. Then as axis $j_{[S_l]}$ is perpendicular to $i_{[S_l]}$ and also system $[S_1]$ is right-hand,

we have for the situation of Figure 2

$$j_{[S_l]} = \frac{k_{[S_l]}\times k_{[G_l]}}{|k_{[S_l]}\times k_{[G_l]}|} = i_{[S_G]}\sin\lambda + j_{[S_G]}(-\cos\lambda) + k_{[S_G]}0$$

Finally, in completing the right-hand system $[S_1]$, we construct

$$i_{[S_l]} = j_{[S_l]}\times k_{[S_l]} = i_{[S_G]}(-\cos\lambda\sin\beta) + j_{[S_G]}(\sin\lambda\sin\beta) + k_{[S_G]}\cos\beta .$$

Then the matrix that yields the components of a vector represented in system $[S_G]$ in terms of its components in system $[S_1]$ is

$$(\sigma_1) = \begin{pmatrix} -\cos\lambda\sin\beta & \sin\lambda & \cos\beta\cos\lambda \\ -\sin\lambda\sin\beta & -\cos\lambda & \cos\beta\sin\lambda \\ \cos\beta & 0 & \sin\beta \end{pmatrix}$$

The remainder of the derivation proceeds in an entirely analogous way from the definition of coordinate system $[S_2]$ described in the opening paragraph of the section labeled "analysis".

Let 1_{SI} be a unit vector in the direction of SI. Then from Figure 2, we have in $[S_1]$ coordinates

$$1_{SI} = i_{[S_1]}\cos\beta_P\cos\alpha_P + j_{[S_1]}\cos\beta_P\sin\alpha_P + k_{[S_1]}(-\sin\beta_P).$$

By definition, the representation of axis $k_{[S_2]}$ in system $[S_1]$ is

$$k_{[S_2]} = -1_{SI} = i_{[S_1]}(-\cos\beta_P\cos\alpha_P) + j_{[S_1]}(-\cos\beta_P\sin\alpha_P) + k_{[S_1]}\sin\beta_P$$

Furthermore, $j_{[S_2]}$ is parallel to $SP\times SI$ and, with 1_{SP} represented in system $[S_1]$ (See Figure 2) as

$$1_{SP} = i_{[S_1]}\cos\alpha_P + j_{[S_1]}\sin\alpha_P + k_{[S_1]}0$$

we obtain

$$j_{[S_2]} = \frac{1_{SP}\times 1_{SI}}{|1_{SP}\times 1_{SI}|} = i_{[S_1]}(-\sin\alpha_P) = j_{[S_1]}(\cos\alpha_P) + k_{[S_1]}0$$

Finally, since $[S_2]$ is to be right-hand,

$$i_{[S_2]} = j_{[S_2]}x\ k_{[S_2]} = i_{[S_1]}\cos\alpha_P\sin\beta_P + j_{[S_1]}\sin\alpha_P\sin\beta_P + k_{[S_1]}\cos\beta_P$$

Thus the matrix (σ_2) from system $[S_2]$ *to system* $[S_1]$ *is*

$$(\sigma_2) = \begin{pmatrix} \cos\alpha_P\sin\beta_P & -\sin\alpha_P & -\cos\alpha_P\cos\beta_P \\ \sin\alpha_P\sin\beta_P & \cos\alpha_P & -\sin\alpha_P\cos\beta_P \\ \cos\beta_P & 0 & \sin\beta_P \end{pmatrix}$$

so that the matrix (M) from $[S_2]$ *to* $[S_G]$ has the form (M) = $(\sigma_1)(\sigma_2)$ With the following elements:

$$m_{11} = -\cos\lambda\sin\beta\cos\alpha_P\sin\beta_P + \sin\lambda\sin\alpha_P\sin\beta_P + \cos\beta\cos\lambda\cos\beta_P$$

$$m_{12} = \cos\lambda\sin\beta\sin\alpha_P + \sin\lambda\cos\alpha_P$$

$$m_{13} = \cos\lambda\sin\beta\cos\alpha_P\cos\beta_P - \sin\lambda\sin\alpha_P\cos\beta_P + \cos\beta\cos\lambda\sin\beta_P$$

$$m_{21} = -\sin\lambda\sin\beta\cos\alpha_P\sin\beta_P - \cos\lambda\sin\alpha_P\sin\beta_P + \cos\beta\sin\lambda\cos\beta_P$$

$$m_{22} = \sin\lambda\sin\beta\sin\alpha_P - \cos\lambda\cos\alpha_P$$

$$m_{23} = \sin\lambda\sin\beta\cos\alpha_P\cos\beta_P + \cos\lambda\sin\alpha_P\cos\beta_P + \cos\beta\sin\lambda\sin\beta_P$$

$$m_{31} = \cos\beta\sin\beta_P\cos\alpha_P + \sin\beta\cos\beta_P$$

$$m32 = -\cos\beta\sin\alpha_P$$

$$m_{33} = -\cos\beta\cos\beta_P\cos\alpha_P + \sin\beta\sin\beta_P$$

APPENDIX 2

DERIVATION OF T_1^{-1}

In order to obtain the translation between points S and I, we proceed as follows. In Figure 5 we are given the point S and are required to find that point I on the earth's surface that translates into S

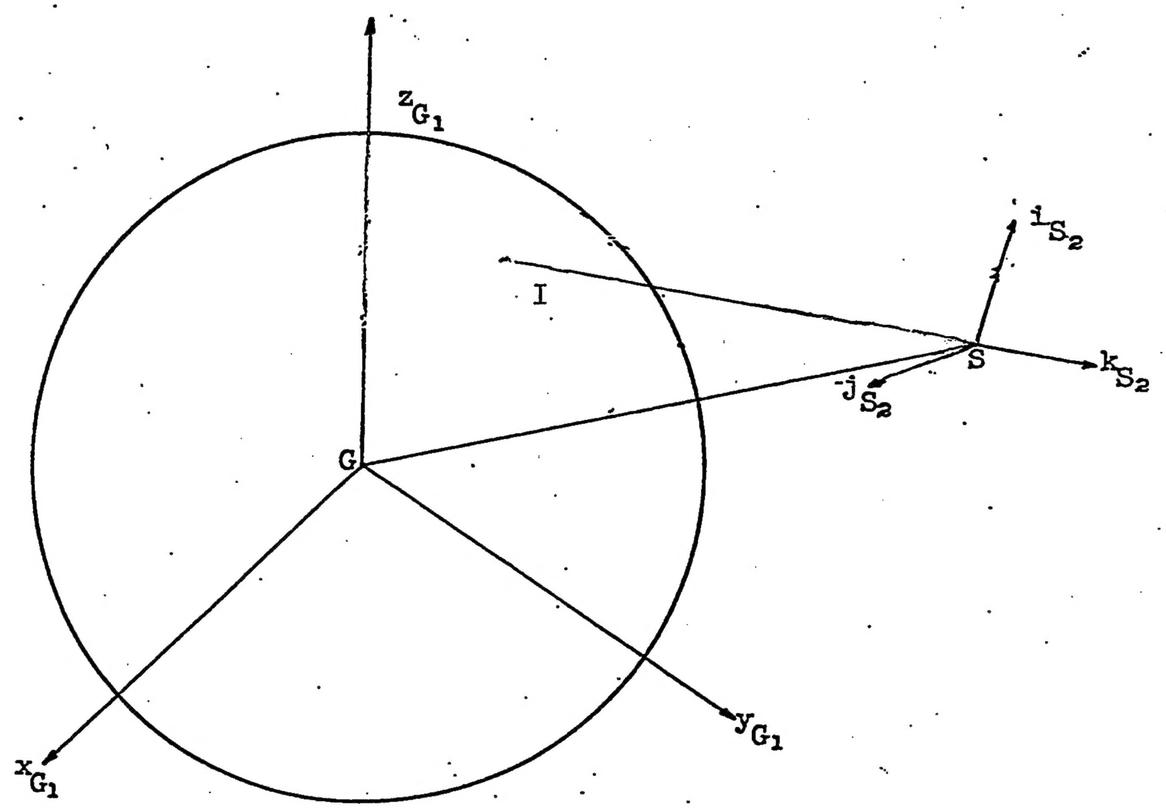

Figure 5 Geometric Relationship Between Points S and I.

In order to do this, we first represent the vector GI (from geocenter to point I) in $[S_2]$ coordinates as $GI_{[S_2]} = GS_{[S_2]} + SI_{[S_2]}$ In system $[S_2]$, GS has the representation

$$GS_{[S_2]} = i_{[S_2]} r\cos\beta_P + j_{[S_2]} 0 + k_{[S_2]} r\sin\beta_P$$

also we have

$$GI_{[S_2]} = i_{[S_2]} r\cos\beta_P + j_{[S_2]} 0 + k_{[S_2]}(-|SI| + r\sin\beta_P)$$

(With the minus sign in front of $|SI|$ because SI is along the negative of the $k_{[S_2]}$ axis) with the value of $|SI|$ to be determined.

As I is on the earth's surface, $|GI|$ = *the radius of the earth* r_e. Inserting this condition and forming $|GI|^2$ we obtain the quadratic

$$|SI|^2 - 2|SI| r\sin\beta_p + r^2 - r_e^2 = 0$$

with the solution

$$|SI| = r\sin\beta_p - \sqrt{r_e^2 - (r\cos\beta_p)^2}$$

where the negative square root has been chosen because it corresponds to the smaller value of $|SI|$ (*since* $0 \leq \beta_P \leq \pi/2$). Figure 5 reveals that the positive square root corresponds to extending line SI through the earth and intersecting it on the other side.

SUMMARY OF THIS REPORT AND OUTLINE FOR WORK STILL TO BE DONE

The development of an equation relating the location "I" of an electromagnetic emitter to a set of variables defining location and orientation of a single satellite sensor system, has been accomplished. Next, the total differential, dI with respect to the variables $r, \lambda, \beta, \beta_p, \alpha_p,$ will be formed as will the function $|dI|^2$. For this last function, the absolute maximums will be determined by examining critical points and domain end-points. This constitutes the work for the single sensor system.

With the pattern of analysis defined by the single-sensor system above, the case of a two-sensor system will add five more variables $r_2, \lambda_2, \beta_2, \alpha_{2_P}, \beta_{2_P}$ and a filtering scheme with which to combine the output of the two sensors. Three or more sensors are handled in an exactly analogous manner as was the two sensor system described above.

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